

Integral Equation Methods In Inverse Scattering

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ABSTRACT

Consider the scattering from a sound soft obstacle (or crack) which can be modelled by the exterior Dirichlet problem for the Helmholtz equation. The task of the direct problem is to find a solution $u^s \in C^2(\mathbb{R}^2 \setminus \bar{D}) \cap C(\mathbb{R}^2 \setminus D)$ to the Helmholtz equation

$$\Delta u^s + k^2 u^s = 0, \quad \text{in } \mathbb{R}^2 \setminus \bar{D} \quad (1)$$

which satisfies the Dirichlet boundary conditions

$$u^s = -u^i \quad \text{on } \Gamma = \partial D \quad (2)$$

and the Sommerfeld radiation condition

$$\lim_{r \rightarrow \infty} \sqrt{r} \left(\frac{\partial u^s}{\partial \nu} - iku^s \right) = 0, \quad r := |x| \quad (3)$$

uniformly for all directions $\hat{x} := \frac{x}{|x|}$ given an incident plane wave $u^i(x, d) := e^{ik\langle x, d \rangle}$ with a wave number $k > 0$ and a unit vector d giving the direction of propagation. This kind of problem is solved successfully using boundary integral equation method, see [1] for example.

Based on the integral equation methods, the solving of the inverse scattering problem leads to a Fredholm integral equation of the first kind

$$F(\Gamma) = u_\infty \quad (4)$$

where Γ is the boundary of the unknown scatterer. Because of the ill-posedness and the non-linearity of this equation, often the regularized Newton's method is used (see [2], [3]).

In this talk we present some variants of nonlinear integral equation method inspired by [4], [5] which transform the inverse problem into a system of two nonlinear integral equations. Several numerical examples will be given at the end of the talk to illustrate our methods.

References

- [1] Colton D and Kress R 1983 *Integral Equation Methods in Scattering Theory* Wiley-Interscience Publication, New York
- [2] Colton D and Kress R 1998 *Inverse Acoustic and Electromagnetic Scattering Theory* 2nd edn (Berlin: Springer)

- [3] Kress R 1995 Integral equation methods in inverse obstacle scattering *Engineering Analysis with Boundary Elements* **15** 171-179
- [4] Kress R and Rundell W 2005 Nonlinear integral equations and the iterative solution for an inverse boundary value problem *Inverse Problems* **21** 1207-1223
- [5] Lee K-M 2006 Inverse scattering via nonlinear integral equations for a Neumann crack *Inverse Problems* **22** 1989-2000